A Channel Update Algorithm for VBLAST Architecture in Vehicular Ad-hoc Networks

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***Abstract*—Vehicular networks require accurate channel state information (CSI) to decode the received signal. Such knowledge is usually obtained via a training sequence. However in vehicular networks, the channel coherence time is very small due to the high speeds of the nodes, therefore the channel estimate from the training is likely to become inaccurate as the decoding proceeds. Using shorter packets can improve the performance at the cost of increased overhead. In this paper we introduce a novel channel tracking algorithm for VBLAST in vehicular networks with relatively little change in the overhead. The algorithm uses first order Kalman filters therefore it has less complexity than available tracking algorithms. The algorithm uses the detected symbols and received signal after the interference cancellation and detection processes of the VBLAST decoder to improve the channel estimation. Simulation results show considerable improvement in mean square error (MSE) and BER when using this algorithm compared to channel estimation by training only with small increase in hardware complexity.**

***Index terms*—Channel Estimation, MIMO, VANET, VBLAST.**

I. INTRODUCTION

The capacity of multiple input multiple output (MIMO) systems was shown to increase with the number of antennas [1]. Several algorithms to achieve part of this capacity have been developed, including space time block codes (STBC), space time trellis codes (STTC) and Bell Labs lAyered Space Time (BLAST) algorithms. Space Time codes increase the reliability of the link making it possible to use higher modulations to achieve higher data rates. BLAST systems, on the other hand, assume the receiver antennas are in a rich Rayleigh fading environment causing each antenna to receive an independent signal.

Vertical-BLAST (VBLAST) makes use of the channel state matrix (**H**) to decode the signal recursively. It starts decoding with the signal that has the highest SNR then cancels its contribution (interference) in the received signal vector, thus achieving better performance than the zero forcing receiver. Other BLAST algorithms exist such as Diagonal, Horizontal and Turbo BLAST but they require more complicated transmitters and/or receivers than VBLAST [2-4].

In VANET vehicles communicate in an ad hoc mode while moving at high speeds, therefore relative speeds of 200km/hr or more between cars in opposite directions are not uncommon. The frequency band allocated for VANET networks is at 5.9GHz leading to a Doppler shift of 1100Hz for 200 km/hr speed, and a channel coherence time of approximately 162μs [5]. When using a training sequence for channel estimation, the short coherence time means a small number of symbols can be transmitted between two training periods thus reducing the bandwidth efficiency due to the large overhead. This is particularly important in VANET since the communication time between the vehicles is very short therefore high data rates are essential to exchange as much information as possible during this small time.

In this paper we introduce an algorithm to update the channel estimation in a flat fading channel. We assume an initial estimate of the channel is available, possibly from a training sequence, and the algorithm enhances this estimation so that longer packets and/or better BER can be achieved. The algorithm can work with any MIMO system but, when combined with VBLAST, can be implemented with a minor increase in hardware complexity. We assume flat fading with known maximum Doppler shift and signal to noise ratio.

The rest of the paper is organised as follows: section II presents some related work. Section III outlines the achievable capacity using VBLAST and is followed by a mathematical derivation of the channel update algorithm. Section V describes the simulations and the results obtained. Finally, section VI summarizes the main contributions of this work.

II. RELATED WORK

Channel estimation has been of interest for many research works. In [6-8] the optimum training sequence for MIMO systems has been investigated. In [6] it was shown that an orthonormal training set is the optimum training sequence for MIMO channels. These can be used to obtain an initial estimate of the channel. In [9] the authors considered the use of Kalman filtering to track the channel for orthogonal STBC MIMO. They exploited the orthogonality of the codes to reduce the complexity of the filter. In [10] a maximum likelihood channel tracking algorithm has been proposed. The authors modelled the channel as an auto regressive (AR) process using Clarke's power spectral density. A combination of Kalman filter and minimum mean square error decision feedback equaliser (MMSE-DFE) was used in [11] to estimate the channel. The DFE is used to estimate the transmitted signal and its output is fed to the Kalman filter for channel estimation. A polynomial fitting is then used to further enhance the channel prediction. In [12] an autoregressive moving average (ARMA) filter was used to model the channel response based on Clarke's channel power spectral density, this was then used to design a Kalman filter for tracking. In this paper we use a bank of first order Kalman filters for channel updating, thus avoiding the computation complexity encountered in these algorithms. The proposed algorithm then recursively estimates the change in the channel and updates the channel matrix to minimise the estimate error, thus improving the BER performance.

III. CAPACITY OF VBLAST

The theoretical capacity (C) of VBLAST has been studied in [13]. It was shown that for a given SNR per receive antenna (ρ) the optimum ratio of the number of transmit to receive antennas (α) is the one that maximises the expression:

$$C \approx \max_{0<\alpha<1} \left\{ \alpha . \log_2 \left[1 + \rho \left(\alpha^{-1} - 1 \right) \right] \right\} \tag{1}$$

Note that the number of transmit antennas is always less than that of receiver antennas to provide diversity. Fig (1) is a comparison between the theoretical capacity, using equation (1), and the capacity obtained from simulations. We assume a 1MHz bandwidth and 3×4 VBLAST system using QAM with perfect channel knowledge at the receiver and optimise the number of transmit antennas, modulation and symbol rate to maximise the capacity while maintaining a maximum bit error rate of 10^{-3} for uncoded data and 10^{-4} when using the 802.11a standard rate ½ convolutional code. The capacity for the optimised values from simulations is given by equation (2).

$$C = \frac{N_t \times Bs \times Rs \times Cr}{N_r \times B} \tag{2}$$

Where N_t is the number of transmit antennas, N_r is the number of receive antennas, *Bs* is the number of bits per symbol, *Rs* is the symbol rate, *Cr* is the code rate and *B* is the bandwidth. These parameters are determined by simulation and are used in (2) to find the optimised capacity plotted in Fig.1.

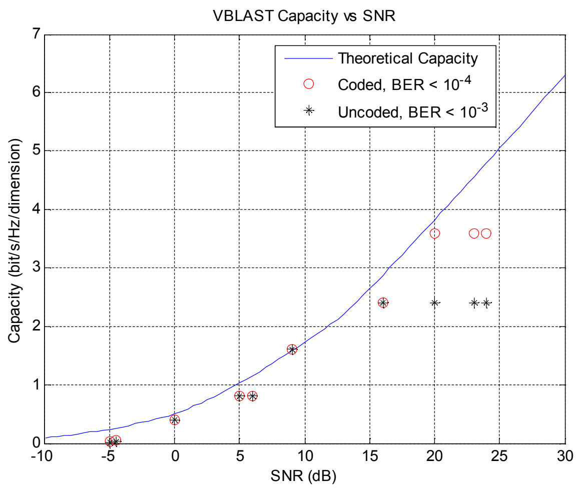

Fig 1. Achievable capacity using 3×4 VBLAST

As can be seen from Fig (1) it is possible to achieve high capacities by using VBLAST. At 20dB a maximum of 2.4bits/s/Hz/dimension is achievable without coding, 3.6bits/s/Hz/dimension with coding compared to the theoretical value of 3.8bits/s/Hz/dimension from equation (1). The results from simulation increase in a staircase manner since the QAM constellation increases in multiples of 2. To achieve this high capacity, however, accurate channel state information matrix is required at the receiver. As the channel varies with time, the channel matrix must be updated frequently to ensure correct decoding. In the next section we develop an algorithm to track the changes in the channel and update the channel matrix at the receiver.

IV. Derivation of the Channel Update Algorithm

For a *p*×*q* VBLAST system with *p* transmit and *q* receive antennas in a flat fading channel, the length *q* column vector of received signal ($\mathbf{r}_{n-1}$) at time index *n-1* can be written as:

$$\mathbf{r}_{n-1} = \mathbf{H}_{n-1}\mathbf{s}_{n-1} + \mathbf{m}_{n-1} \tag{3}$$

Here $\mathbf{H}_{n-1}$ is the $q\times p$ channel matrix, $\mathbf{s}_{n-1}$ is the *p* column vector of transmitted symbols and $\mathbf{m}_{n-1}$ is the *q* column vector of white noise at time *n-1*. Throughout this paper, lower and upper case bold characters represent vectors and matrices respectively while lower case characters represent elements within the matrix/vector while $(.)^+$ represents the Moore-Penrose pseudo inverse process.

Let the estimated channel matrix be $\hat{\mathbf{H}}_{n-1}$. The simplest BLAST receiver (zero forcing receiver) calculates an estimate of the transmitted symbols ($\hat{\mathbf{s}}_{n-1}$) using the pseudo inverse of the channel matrix ($\hat{\mathbf{H}}_{n-1}^{+}$) as:

$$\hat{\mathbf{s}}_{n-1} = \hat{\mathbf{H}}_{n-1}^{+} \times \mathbf{r}_{n-1} \tag{4}$$

since for a full rank $q\times p$, $p\leq q$ matrix $\mathbf{H}$ we have [14]:

$$\mathbf{H}^+\mathbf{H} = \mathbf{I_p} \tag{5}$$

$\mathbf{I_p}$ is the $p\times p$ identity matrix. Define $\Delta\mathbf{H}_n$ as:

$$\Delta\mathbf{H}_n = \left(\mathbf{r}_{n-1} - \hat{\mathbf{H}}_{n-1}\hat{\mathbf{s}}_{n-1}\right)\times\hat{\mathbf{s}}_{n-1}^{+} \tag{6}$$

Substituting equation (3) in (6) and assuming correct decoding ($\mathbf{s}_{n-1} = \hat{\mathbf{s}}_{n-1}$) we find:

$$\Delta\mathbf{H}_n = \left(\mathbf{H}_{n-1} - \hat{\mathbf{H}}_{n-1}\right)\times\mathbf{s}_{n-1}\mathbf{s}_{n-1}^{+} + \mathbf{m}_{n-1}\mathbf{s}_{n-1}^{+} \tag{7}$$

Note that the term $\left(\mathbf{r}_{n-1} - \hat{\mathbf{H}}_{n-1}\hat{\mathbf{s}}_{n-1}\right)$ is calculated in the cancellation step of the VBLAST decoding algorithm. $\Delta\mathbf{H}_n$ can be used with a simple first order Kalman filter to improve the channel estimation as:

$$\hat{\mathbf{H}}_n = \hat{\mathbf{H}}_{n-1} + \mathbf{K}.\Delta\mathbf{H}_n \tag{8}$$

Where $\mathbf{K}$ is a matrix of update parameters and the dot in (8) represents element by element multiplication.

We now need to find the optimum value of $\mathbf{K}$, however since we assume the receive antennas are not correlated; we need to optimise for only one antenna. Equation (7) can be rewritten for the elements of the matrix $\Delta\mathbf{H}_n$ as:

$$\Delta h_{ij}^{\ n} = \left(r_i^{\ n-1} - \sum_{l=1}^{p} \hat{h}_{il}^{\ n-1}.\hat{s}_l^{\ n-1} \right) a_j^{\ n-1} \tag{9}$$

The lower case character represents elements of the matrix/vector denoted by upper/lower case bold character. The subscripts identify the row (*i*) and column (*j* or *l*) which represent receive and transmit antennas respectively while the superscript (*n*) denotes the time index. a_j represents the element at column *j* of the row vector ($\hat{\mathbf{s}}^+$). Equation (9) can be expanded using (3) as:

$$\Delta h_{ij}^{\ n} = \left(\sum_{l=1}^{p} \left(h_{il}^{\ n-1}.s_l^{\ n-1} - \hat{h}_{il}^{\ n-1}.\hat{s}_l^{\ n-1} + m_i^{\ n-1} \right) \right) a_j^{\ n-1} \tag{10}$$

and assuming correct decoding:

$$\begin{aligned}\Delta h_{ij}^{\ n} &= \left(\sum_{l=1}^{p} \left(h_{il}^{\ n-1} - \hat{h}_{il}^{\ n-1} \right) s_l^{\ n-1} \right) a_j^{\ n-1} + m_i^{\ n-1} a_j^{\ n-1} \\ &= \beta\varepsilon_{ij}^{\ n-1} + \sum_{\substack{l=1\\ l\neq j}}^{p} \varepsilon_{il}^{\ n-1}.s_l^{\ n-1}.a_j^{\ n-1} + m_i^{\ n-1} a_j^{\ n-1}\end{aligned} \tag{11}$$

Where $\varepsilon_{ij}^{n-1} = h_{ij}^{n-1} - \hat{h}_{ij}^{n-1}$ and β is the product of the s_j^{n-1} and a_j^{n-1} terms [14]. The elements of the updated channel can be written as:

$$\hat{h}_{ij}^{\ n} = \hat{h}_{ij}^{\ n-1} + k_{ij}\Delta h_{ij}^{\ n} \quad (12)$$

$$\hat{h}_{ij}^{\ n} = \hat{h}_{ij}^{\ n-1} + \beta k_{ij}\varepsilon_{ij}^{\ n-1} + k_{ij}\sum_{\substack{l=1\\ l\neq j}}^{p}\varepsilon_{il}^{\ n-1}s_{l}^{\ n-1}a_{j}^{\ n-1} + k_{ij}m_{i}^{\ n-1}a_{j}^{\ n-1} \quad (13)$$

An analysis of the probability density function of the third term of (13) shows that it is approximately Gaussian. Therefore the last two terms in (13) can be approximated by white noise with average power [15]:

$$\overline{N}_{0,j} = \frac{N_0}{\rho_j}\left(1 + \sum_{\substack{l=1\\ l\neq j}}^{p} e_l\right) \quad (14)$$

Where N_0 is the original total white noise power for the receive antenna i, e_l is the average error covariance reduction value and ρ_j is a constant that specifies the fraction of noise associated with stream j. The optimum value of k_{ij} is the one that minimises the value $\sigma^2 = E\left[\left|h_{ij}^{\ n} - \hat{h}_{ij}^{\ n}\right|^2\right]$.

In our derivation of the optimum **K** parameters we adopt Clarke's power spectrum density ($P(f)$) defined for a maximum Doppler shift f_D as [16]:

$$P(f) = \begin{cases} \dfrac{1}{\pi f_D}\dfrac{1}{\sqrt{1-\left(\dfrac{f}{f_D}\right)^2}}, & |f| < f_D \\ 0, & otherwise \end{cases} \quad (15)$$

We calculate the optimum set of **K** parameters by differentiating σ^2 with respect to k_j and setting the derivative equal to zero. After some lengthy but straight forward mathematical manipulation and assuming the receiver antennas are uncorrelated with equal average SNR, the optimum set of **K** parameters is given by:

$$k_{ij} = k_j \quad \forall i \quad (16)$$

$$k_j = 3.6\sqrt[3]{\frac{\rho_j (f_D T_s)^2}{\beta N_0 (1 + \sum_{\substack{l=1\\ l\neq j}}^{p} e_l)}} = 3.6\sqrt[3]{\frac{(f_D T_s)^2}{N_0 (1 + \sum_{\substack{l=1\\ l\neq j}}^{p} e_l)}} \quad (17)$$

$$e_j \approx \frac{0.75}{p}k_j \quad (18)$$

$$N_0 = \frac{1}{E_s/N_0} \quad (19)$$

Where T_s is the symbol duration.

We define E_s/N_0 as the total SNR if all transmitting antennas transmit the same symbol. In (17) β is equal to $1/p$ [14] and we set and ρ_j equal to $1/p$ since we assume equal average transmit (receive) power for each transmit (receive) antenna. The k_j parameters are calculated recursively. First we assume no interference from the other symbols and set $e_j = 0$. We then calculate k_1 and update e_1. Next we substitute the new value of e_1 for k_2 then update e_2. This process is repeated till all the k_j and e_j parameters are calculated and then we repeat the calculations again. This process converges very quickly and the final values of k_j are not very different from the initial ones. The k_j parameters then can be used to update the channel estimate. The algorithm requires the calculation of p k_j parameters, one for each transmit antenna using equations (18) and (19). These can be calculated once at the beginning of the packet and held constant for the duration of the packet. $\mathbf{\Delta H}_n$ requires the pseudo inverse of the ($p\times 1$) vector $\mathbf{s}$, which can be pre-computed and stored, and then multiplying it by the term $(\mathbf{r}_{n-1} - \hat{\mathbf{H}}_{n-1}\hat{\mathbf{s}}_{n-1})$, equation (6), which is calculated in the VBLAST algorithm. This multiplication consists of $p\times q$ complex multiplications. The channel update, equation (8), requires $p\times q$ real by complex multiplications and $p\times q$ complex additions.

CALCULATION OF k_j PARAMETERS ALGORITHM
1) set $e_j = 0$ for all j
2) iteration = 1
3) j = 1
4) calculate k_j using equation (17)
5) calculate e_j using equation (18)
6) j = j+1
7) if (j < number of transmit antennas) go to 4
8) iteration = iteration +1
9) if (iteration < max number of iterations) go to 3

Table 1. Calculation of k_j parameters algorithm

CHANNEL UPDATE ALGORITHM
1) calculate the k_j parameters
2) calculate $\mathbf{\Delta H}$ *using equation (6)*
3) update the channel using equation (8)

Table 2. Channel update algorithm

V. SIMULATION MODEL AND RESULTS

Numerous channel models to simulate wireless channels exist [17-20] but the ring model is the most common. The ring model was designed to simulate mobile-basestation links with dense environment around the mobile terminal. A two ring model was proposed in [17] for vehicular networks, however, it is not realistic for cars on motorways since the number of surroundings will be small. Instead we use the elliptical model proposed in [20] and shown in Fig (2) modified for the high speed nodes.

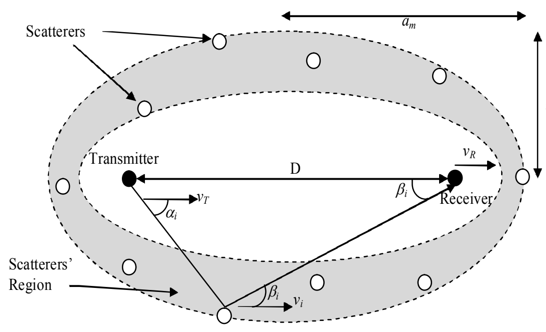

Fig 2. Elliptical Channel model

The dimensions of the ellipse can be calculated from the delay spread of the channel [19]. In [21-22] the delay spread for VANET was measured for the city and on highways and the minimum mean delay spread was 109ns. We adopt this value in our model since as the delay spread increases the distribution of the angle of arrival (AOA) at the receiver approaches uniform distribution in $[0, 2\pi)$ which is ideal for VBLAST since low correlation between the antennas can be achieved [23]. We further assume no line of sight exists, due to cars between the communicating nodes, and the distance is 1km.

We ran a number of simulations using Matlab to study the performance of the algorithm. In our simulations we use a 2×4 VBLAST system 1MSymbol/s, 5.9GHz and the channel model shown in Fig (2). In the simulations, initially the algorithm will have perfect channel knowledge rather than estimating from a training sequence. This is necessary to isolate any errors that might arise from the use of training sequence estimation. We use the k_j values calculated from only 1 iteration to reduce the complexity. Fig (3) shows the MSE in the estimated channel for the cases of 256, 512 and 1024 symbols per antenna using QPSK modulation with channel update, using equation (7) and (16) to (18), compared to 256 without update. As can be seen from Fig (2) the update algorithm reduces the MSE by 50% at 12dB Es/N_0. The MSE in Fig (3) without update does not depend on the SNR because the receiver is assumed to have perfect, noise free, estimate of the channel at the beginning of the packet and this is held constant for the duration of the packet. Fig (4) shows the MSE vs. the symbol number for 26dB E_s/N_0. Initially the receiver will have perfect channel knowledge (MSE $\approx$ 0) but with time this estimate becomes invalid due to the high Doppler shift. Fig (5) shows the BER performance of QPSK for various relative vehicle speeds. As can be seen the performance improves considerably when the algorithm is used and is 2dB from that of perfect channel knowledge for 60km/hr.

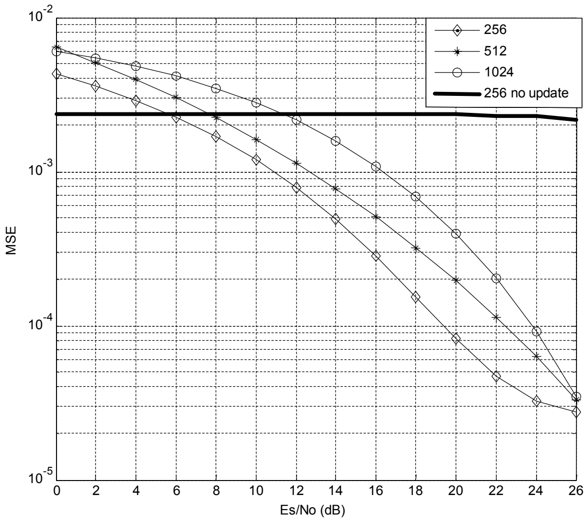

Fig 3. MSE of Channel Estimation for 180 km/h

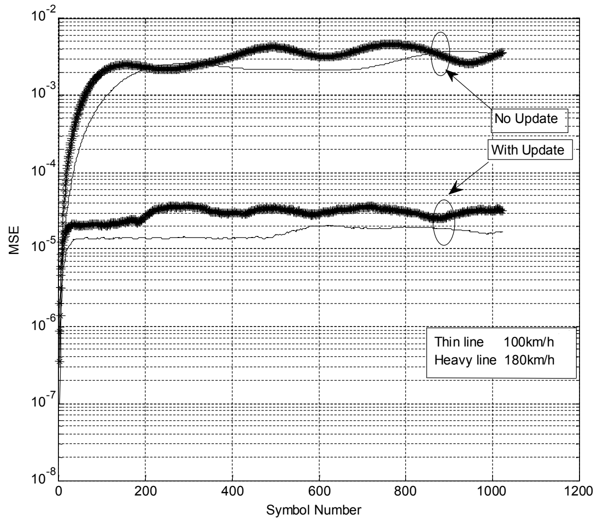

Fig 4. MSE of Channel Estimation vs. No of symbols

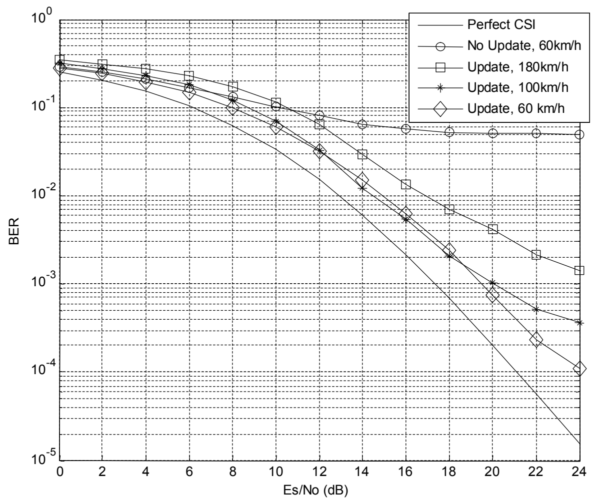

Fig 5. QPSK BER with and without channel update

Fig (6) shows the performance of the same system using QPSK with various packet lengths for a speed of 60 km/hr. As can be seen from the figure, the performance degrades as the packet length increases; this is due to two reasons. The first reason is estimation error, as the estimation process proceeds, the error in the estimation accumulates and for long packets this will lead to erroneous results near the end of the packet. The second reason is detection errors since the probability of symbol errors increases as the packet length increases. The estimation algorithm assumes correct decoding; therefore such errors will affect the performance of the algorithm.

VI. CONCLUSION

In this paper we developed a simple recursive algorithm to keep track of changes in the channel and to update the channel estimation matrix for VBLAST. The update algorithm enhances the channel estimation on a symbol by symbol basis, but this can be relaxed for high symbol rates and/or slow fading as the channel coherence time will be large compared to the symbol duration. The proposed algorithm improves system BER and channel estimate MSE via continuous and accurate channel updating and has less computational complexity compared to existing tracking algorithms as a result of using a simplified Kalman filter. Simulation results showed remarkable improvements when using the update algorithm compared to the training only channel estimation. The algorithm is capable of updating the channel estimation for VBLAST for nodes moving at high speeds thus improving the bit error rate of VANET. Further work is going on to extend the algorithm to frequency selective fading and OFDM.

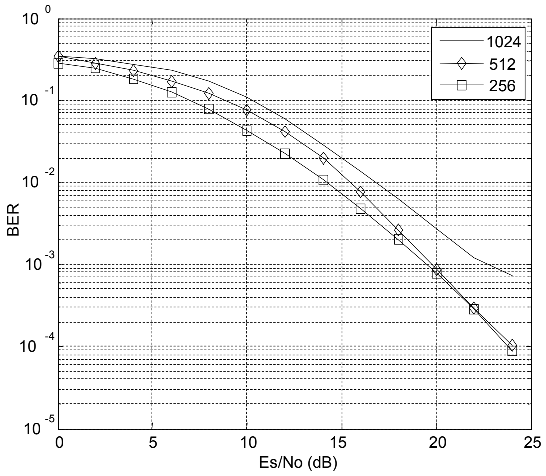

Fig 6. BER for different packet sizes, 60 km/h

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